CHAPTER 17 | THE PRINCIPLE OF LINEAR SUPERPOSITION AND INTERFERENCE PHENOMENA

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (d) If we add pulses 1 and 4 as per the principle of linear superposition, the resultant is a straight horizontal line that extends across the entire graph.

2. (a) These two pulses combine to produce a peak that is 4 units high and a valley that is 2 units deep. No other combination gives greater values.

3. (c) The smallest difference in path lengths for destructive interference to occur is one-half a wavelength \( \frac{1}{2} \lambda \). As the frequency goes up, the wavelength goes down, so the separation between the cellists decreases.

4. Smallest separation = 1.56 m

5. (b) According to Equation 17.2, the diffraction angle \( \theta \) is related to the wavelength and diameter by \( \sin \theta = 1.22 \left( \frac{\lambda}{D} \right) \) and is determined by the ratio \( \lambda/D \). Here the ratio is \( 2\lambda_0/D_0 \) and is the largest of any of the choices, so it yields the largest diffraction angle.

6. \( \theta = 30.0 \) degrees

7. (e) Since the wavelength is directly proportional to the speed of the sound wave (see Section 16.2), the wavelength is greatest in the helium-filled room. The greater the wavelength, the greater the diffraction angle \( \theta \) (see Section 17.3). Thus, the greatest diffraction occurs in the helium-filled room.

8. (d) The trombones produce 6 beats every 2 seconds, so the beat frequency is 3 Hz. The second trombone can be producing a sound whose frequency is either 438 Hz – 3 Hz = 435 Hz or 438 Hz + 3 Hz = 441 Hz.

9. Beat frequency = 3.0 Hz

10. (d) According to the discussion in Section 17.5, one loop of a transverse standing wave corresponds to one-half a wavelength. The two loops in the top picture mean that the wavelength of 1.2 m is also the distance \( L \) between the walls, so \( L = 1.2 \) m. The bottom picture contains three loops in a distance of 1.2 m, so its wavelength is \( \frac{2}{3} (1.2 \text{ m}) = 0.8 \text{ m} \).
11. (b) The frequency of a standing wave is directly proportional to the speed of the traveling waves that form it (see Equation 17.3). The speed of the waves, on the other hand, depends on the mass $m$ of the string through the relation $v = \sqrt{F / (m/L)}$, so the smaller the mass, the greater is the speed and, hence, the greater the frequency of the standing wave.

12. (c) For a string with a fixed length, tension, and linear density, the frequency increases when the harmonic number $n$ increases from 4 to 5 (see Equation 17.3). According to $\lambda = \nu f$ (Equation 16.1), the wavelength decreases when the frequency increases.

13. Fundamental frequency $= 2.50 \times 10^2$ Hz

14. (c) One loop of a longitudinal standing wave corresponds to one-half a wavelength. Since this standing wave has two loops, its wavelength is equal to the length of the tube, or 0.80 m.

15. (b) There are two loops in this longitudinal standing wave. This means that the 2$^{nd}$ harmonic is being generated. According to Equation 17.4, the $n^{th}$ harmonic frequency is $f_n = n \left( \frac{\nu}{2L} \right)$, where $\frac{\nu}{2L}$ is the fundamental frequency. Since $f_2 = 440$ Hz and $n = 2$, we have $\frac{\nu}{2L} = \frac{440 \text{ Hz}}{2} = 220$ Hz.

16. (c) The standing wave pattern in the drawing corresponds to $n = 3$ (the 3$^{rd}$ harmonic) for a tube open at only one end. Using Equation 17.5, the length of the tube is $L = \frac{n \nu}{4f_n} = \frac{(3)(343 \text{ m/s})}{4(660 \text{ Hz})} = 0.39$ m.

17. Frequency of 3$^{rd}$ harmonic $= 9.90 \times 10^2$ Hz
7. **SSM REASONING** The geometry of the positions of the loudspeakers and the listener is shown in the following drawing.

The listener at C will hear either a loud sound or no sound, depending upon whether the interference occurring at C is constructive or destructive. If the listener hears no sound, destructive interference occurs, so

\[ d_2 - d_1 = \frac{n\lambda}{2} \quad n = 1, 3, 5, \ldots \]  

**(1)**

**SOLUTION** Since \( v = \lambda f \), according to Equation 16.1, the wavelength of the tone is

\[ \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{68.6 \text{ Hz}} = 5.00 \text{ m} \]

Speaker B will be closest to Speaker A when \( n = 1 \) in Equation (1) above, so

\[ d_2 = \frac{n\lambda}{2} + d_1 = \frac{5.00 \text{ m}}{2} + 1.00 \text{ m} = 3.50 \text{ m} \]

From the figure above we have that,

\[ x_1 = (1.00 \text{ m}) \cos 60.0^\circ = 0.500 \text{ m} \]

\[ y = (1.00 \text{ m}) \sin 60.0^\circ = 0.866 \text{ m} \]
Then

\[ x_2^2 + y^2 = d_2^2 = (3.50 \text{ m})^2 \quad \text{or} \quad x_2 = \sqrt{(3.50 \text{ m})^2 - (0.866 \text{ m})^2} = 3.39 \text{ m} \]

Therefore, the closest that speaker A can be to speaker B so that the listener hears no sound is \( x_1 + x_2 = 0.500 \text{ m} + 3.39 \text{ m} = \boxed{3.89 \text{ m}} \).

8. **REASONING** The two speakers are vibrating exactly out of phase. This means that the conditions for constructive and destructive interference are opposite of those that apply when the speakers vibrate in phase, as they do in Example 1 in the text. Thus, for two wave sources vibrating exactly out of phase, a difference in path lengths that is zero or an integer number \((1, 2, 3, \ldots)\) of wavelengths leads to destructive interference; a difference in path lengths that is a half-integer number \(\left(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\right)\) of wavelengths leads to constructive interference. First, we will determine the wavelength being produced by the speakers. Then, we will determine the difference in path lengths between the speakers and the observer and compare the differences to the wavelength in order to decide which type of interference occurs.

**SOLUTION** According to Equation 16.1, the wavelength \(\lambda\) is related to the speed \(v\) and frequency \(f\) of the sound as follows:

\[ \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{429 \text{ Hz}} = 0.800 \text{ m} \]

Since ABC in Figure 17.7 is a right triangle, the Pythagorean theorem applies and the difference \(\Delta d\) in the path lengths is given by

\[ \Delta d = d_{AC} - d_{BC} = \sqrt{d_{AB}^2 + d_{BC}^2} - d_{BC} \]

We will now apply this expression for parts (a) and (b).

a. When \(d_{BC} = 1.15 \text{ m}\), we have

\[ \Delta d = \sqrt{d_{AB}^2 + d_{BC}^2} - d_{BC} = \sqrt{(2.50 \text{ m})^2 + (1.15 \text{ m})^2} - 1.15 \text{ m} = 1.60 \text{ m} \]

Since \(1.60 \text{ m} = 2(0.800 \text{ m}) = 2\lambda\), it follows that the interference is \boxed{destructive} (the speakers vibrate out of phase).

b. When \(d_{BC} = 2.00 \text{ m}\), we have

\[ \Delta d = \sqrt{d_{AB}^2 + d_{BC}^2} - d_{BC} = \sqrt{(2.50 \text{ m})^2 + (2.00 \text{ m})^2} - 2.00 \text{ m} = 1.20 \text{ m} \]
Since \( 1.20 \text{ m} = 1.5(0.800 \text{ m}) = \left(\frac{3}{2}\right)\lambda \), it follows that the interference is constructive (the speakers vibrate out of phase).

10. **REASONING** When the listener is standing midway between the speakers, both sound waves travel the same distance from the speakers to the listener. Since the speakers are vibrating out of phase, when the diaphragm of one speaker is moving outward (creating a condensation), the diaphragm of the other speaker is moving inward (creating a rarefaction). Whenever a condensation from one speaker reaches the listener, it is met by a rarefaction from the other, and vice versa. Therefore, the two sound waves produce destructive interference, and the listener hears no sound.

When the listener begins to move sideways, the distance between the listener and each speaker is no longer the same. Consequently, the sound waves no longer produce destructive interference, and the sound intensity begins to increase. When the difference in path lengths \( \ell_1 - \ell_2 \) traveled by the two sounds is one-half a wavelength, or \( \ell_1 - \ell_2 = \frac{1}{2} \lambda \), constructive interference occurs, and a loud sound will be heard.

**SOLUTION** The two speakers are vibrating out of phase. Therefore, when the difference in path lengths \( \ell_1 - \ell_2 \) traveled by the two sounds is one-half a wavelength, or \( \ell_1 - \ell_2 = \frac{1}{2} \lambda \), constructive interference occurs. Note that this condition is different than that for two speakers vibrating in phase. The frequency \( f \) of the sound is equal to the speed \( v \) of sound divided by the wavelength \( \lambda \); \( f = \frac{v}{\lambda} \) (Equation 16.1). Thus, we have that

\[
\ell_1 - \ell_2 = \frac{1}{2} \lambda = \frac{v}{2f} \quad \text{or} \quad f = \frac{v}{2(\ell_1 - \ell_2)}
\]

The distances \( \ell_1 \) and \( \ell_2 \) can be determined by applying the Pythagorean theorem to the right triangles in the drawing:

\[
\ell_1 = \sqrt{(4.00 \text{ m})^2 + (1.50 \text{ m} + 0.92 \text{ m})^2} = 4.68 \text{ m}
\]

\[
\ell_2 = \sqrt{(4.00 \text{ m})^2 + (1.50 \text{ m} - 0.92 \text{ m})^2} = 4.04 \text{ m}
\]

The frequency of the sound is

\[
f = \frac{v}{2(\ell_1 - \ell_2)} = \frac{343 \text{ m/s}}{2(4.68 \text{ m} - 4.04 \text{ m})} = 270 \text{ Hz}
\]
19. **REASONING** The beat frequency of two sound waves is the difference between the two sound frequencies. From the graphs, we see that the period of the wave in the upper text figure is 0.020 s, so its frequency is $f_1 = 1/T_1 = 1/(0.020 \text{ s}) = 5.0 \times 10^1 \text{ Hz}$. The frequency of the wave in the lower figure is $f_2 = 1/(0.024 \text{ s}) = 4.2 \times 10^1 \text{ Hz}$.

**SOLUTION** The beat frequency of the two sound waves is

$$f_{\text{beat}} = f_1 - f_2 = 5.0 \times 10^1 \text{ Hz} - 4.2 \times 10^1 \text{ Hz} = 8 \text{ Hz}$$

23. **REASONING** When two frequencies are sounded simultaneously, the beat frequency produced is the difference between the two. Thus, knowing the beat frequency between the tuning fork and one flute tone tells us only the difference between the known frequency and the tuning-fork frequency. It does not tell us whether the tuning-fork frequency is greater or smaller than the known frequency. However, two different beat frequencies and two flute frequencies are given. Consideration of both beat frequencies will enable us to find the tuning-fork frequency.

**SOLUTION** The fact that a 1-Hz beat frequency is heard when the tuning fork is sounded along with the 262-Hz tone implies that the tuning-fork frequency is either 263 Hz or 261 Hz. We can eliminate one of these values by considering the fact that a 3-Hz beat frequency is heard when the tuning fork is sounded along with the 266-Hz tone. This implies that the tuning-fork frequency is either 269 Hz or 263 Hz. Thus, the tuning-fork frequency must be 263 Hz.

33. **REASONING** A standing wave is composed of two oppositely traveling waves. The speed $\nu$ of these waves is given by $\nu = \sqrt{\frac{F}{m/L}}$ (Equation 16.2), where $F$ is the tension in the string and $m/L$ is its linear density (mass per unit length). Both $F$ and $m/L$ are given in the statement of the problem. The wavelength $\lambda$ of the waves can be obtained by visually inspecting the standing wave pattern. The frequency of the waves is related to the speed of the waves and their wavelength by $f = \nu/\lambda$ (Equation 16.1).

**SOLUTION**

a. The speed of the waves is

$$\nu = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{280 \text{ N}}{8.5 \times 10^{-3} \text{ kg/m}}} = 180 \text{ m/s}$$

b. Two loops of any standing wave comprise one wavelength. Since the string is 1.8 m long and consists of three loops (see the drawing), the wavelength is

1.8 m
\[ \lambda = \frac{2}{3} (1.8 \text{ m}) = 1.2 \text{ m} \]

c. The frequency of the waves is
\[ f = \frac{v}{\lambda} = \frac{180 \text{ m/s}}{1.2 \text{ m}} = 150 \text{ Hz} \]

35. **REASONING** The fundamental frequency \( f_1 \) of the wire is given by \( f_1 = \frac{v}{(2L)} \) (Equation 17.3, with \( n = 1 \)), where \( v \) is the speed at which the waves travel on the wire and \( L \) is the length of the wire. The speed is related to the tension \( F \) in the wire according to \( v = \sqrt{\frac{F}{m/L}} \) (Equation 16.2), where \( m/L \) is the mass per unit length of the wire.

The tension in the wire in Part 2 of the text drawing is less than the tension in Part 1. The reason is related to Archimedes’ principle (see Equation 11.6). This principle indicates that when an object is immersed in a fluid, the fluid exerts an upward buoyant force on the object. In Part 2 the upward buoyant force from the mercury supports part of the block’s weight, thus reducing the amount of the weight that the wire must support.

**SOLUTION** Substituting Equation 16.2 into Equation 17.3, we can obtain the fundamental frequency of the wire:

\[ f_1 = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{F}{m/L}} \]

In Part 1 of the text drawing, the tension \( F \) balances the weight of the block, keeping it from falling. The weight of the block is its mass times the acceleration due to gravity (see Equation 4.5). The mass, according to Equation 11.1 is the density \( \rho_{\text{copper}} \) times the volume \( V \) of the block. Thus, the tension in Part 1 is

**Part 1 tension** \[ F = (\text{mass}) \, g = \rho_{\text{copper}} \, V \, g \]

In Part 2 of the text drawing, the tension is reduced from this amount by the amount of the upward buoyant force. According to Archimedes’ principle, the buoyant force is the weight of the liquid mercury displaced by the block. Since half of the block’s volume is immersed, the volume of mercury displaced is \( \frac{1}{2} V \). The weight of this mercury is the mass times the acceleration due to gravity. Once again, according to Equation 11.1, the mass is the density \( \rho_{\text{mercury}} \) times the volume, which is \( \frac{1}{2} V \). Thus, the tension in Part 2 is

**Part 2 tension** \[ F = \rho_{\text{copper}} \, V \, g - \rho_{\text{mercury}} \left( \frac{1}{2} V \right) \, g \]

With these two values for the tension we can apply Equation (1) to both parts of the drawing and obtain
36. **REASONING** The frequencies $f_n$ of the standing waves allowed on a string fixed at both ends are given by Equation 17.3 as $f_n = n \left( \frac{v}{2L} \right)$, where $n$ is an integer that specifies the harmonic number, $v$ is the speed of the traveling waves that make up the standing waves, and $L$ is the length of the string. The speed $v$ is related to the tension $F$ in the string and the linear density $m/L$ via $v = \sqrt{\frac{F}{m/L}}$ (Equation 16.2). Therefore, the frequencies of the standing waves can be written as

$$f_n = n \left( \frac{v}{2L} \right) = n \left( \sqrt{\frac{F}{m/L}} \right) = \frac{n}{2L} \sqrt{\frac{F}{m/L}}$$

The tension $F$ in each string is provided by the weight $W$ (either $W_A$ or $W_B$) that hangs from the right end, so $F = W$. Thus, the expression for $f_n$ becomes $f_n = \frac{n}{2L} \sqrt{\frac{W}{m/L}}$. We will use this relation to find the weight $W_B$. 

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**Part 1**

$$f_1 = \frac{1}{2L} \sqrt{\frac{\rho_{\text{copper}} Vg}{m/L}}$$

**Part 2**

$$f_1 = \frac{1}{2L} \sqrt{\frac{\rho_{\text{copper}} Vg - \rho_{\text{mercury}} \left( \frac{1}{2} V \right) g}{m/L}}$$

Dividing the fundamental frequency of Part 2 by that of Part 1 gives

$$\frac{f_1, \text{ Part 2}}{f_1, \text{ Part 1}} = \frac{\frac{1}{2L} \sqrt{\frac{\rho_{\text{copper}} Vg - \rho_{\text{mercury}} \left( \frac{1}{2} V \right) g}{m/L}}}{\frac{1}{2L} \sqrt{\frac{\rho_{\text{copper}} Vg}{m/L}}} = \sqrt{\frac{\rho_{\text{copper}} - \frac{1}{2} \rho_{\text{mercury}}}{\rho_{\text{copper}}}}$$

$$= \sqrt{\frac{8890 \text{ kg/m}^3 - \frac{1}{2} \left( 13600 \text{ kg/m}^3 \right)}{8890 \text{ kg/m}^3}} = 0.485$$
**Solution** String A has one loop so \( n = 1 \), and the frequency \( f_1^A \) of this standing wave is 
\[
f_1^A = \frac{1}{2L} \sqrt{\frac{W_A}{m/L}}.
\]
String B has two loops so \( n = 2 \), and the frequency \( f_2^B \) of this standing wave is  
\[
f_2^B = \frac{2}{2L} \sqrt{\frac{W_B}{m/L}}.
\]
We are given that the two frequencies are equal, so
\[
\frac{1}{2L} \sqrt{\frac{W_A}{m/L}} = \frac{2}{2L} \sqrt{\frac{W_B}{m/L}}
\]
Solving this expression for \( W_B \) gives
\[
W_B = \frac{1}{4} W_A = \frac{1}{4} (44 \text{ N}) = 11 \text{ N}
\]

**Reasoning** We can find the extra length that the D-tuner adds to the E-string by calculating the length of the D-string and then subtracting from it the length of the E-string.

For standing waves on a string that is fixed at both ends, Equation 17.3 gives the frequencies as 
\[
f_n = \frac{n(v/2L)}{m/L}.
\]
The ratio of the fundamental frequency of the D-string to that of the E-string is
\[
\frac{f_D}{f_E} = \frac{v/(2L)}{v/(2L_E)} = \frac{L_E}{L_D}
\]
This expression can be solved for the length \( L_D \) of the D-string in terms of quantities given in the problem statement.

**Solution** The length of the D-string is  
\[
L_D = L_E \left( \frac{f_E}{f_D} \right) = (0.628 \text{ m}) \left( \frac{41.2 \text{ Hz}}{36.7 \text{ Hz}} \right) = 0.705 \text{ m}
\]
The length of the E-string is extended by the D-tuner by an amount  
\[
L_D - L_E = 0.705 \text{ m} - 0.628 \text{ m} = 0.077 \text{ m}
\]

**Reasoning and Solution** We know that \( L = v/(2f) \). For 20.0 Hz
\[
L = (343 \text{ m/s})/[2(20.0 \text{ Hz})] = 8.6 \text{ m}
\]
For 20.0 kHz
\[ L = \frac{(343 \text{ m/s})}{[2(20.0 \times 10^3 \text{ Hz})]} = 8.6 \times 10^{-3} \text{ m} \]

52. **REASONING AND SOLUTION** The original tube has a fundamental given by \( f = \frac{v}{4L} \), so that its length is \( L = \frac{v}{4f} \). The cut tube that has one end closed has a length of \( L_c = \frac{v}{4f_c} \), while the cut tube that has both ends open has a length \( L_o = \frac{v}{2f_o} \).

We know that \( L = L_c + L_o \). Substituting the expressions for the lengths and solving for \( f \) gives

\[
\frac{f_o f_c}{2f_c + f_o} = \frac{(425 \text{ Hz})(675 \text{ Hz})}{2(675 \text{ Hz}) + 425 \text{ Hz}} = 162 \text{ Hz}
\]

61. **REASONING** When the difference \( \ell_1 - \ell_2 \) in path lengths traveled by the two sound waves is a half-integer number \( \left( \frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, \ldots \right) \) of wavelengths, destructive interference occurs at the listener. When the difference in path lengths is zero or an integer number \( (1, 2, 3, \ldots) \) of wavelengths, constructive interference occurs. Therefore, we will divide the distance \( \ell_1 - \ell_2 \) by the wavelength of the sound to determine if constructive or destructive interference occurs. The wavelength is, according to Equation 16.1, the speed \( v \) of sound divided by the frequency \( f \); \( \lambda = \frac{v}{f} \).

**SOLUTION**

a. The distances \( \ell_1 \) and \( \ell_2 \) can be determined by applying the Pythagorean theorem to the two right triangles in the drawing:

\[
\ell_1 = \sqrt{(2.200 \text{ m})^2 + (1.813 \text{ m})^2} = 2.851 \text{ m}
\]

\[
\ell_2 = \sqrt{(2.200 \text{ m})^2 + (1.187 \text{ m})^2} = 2.500 \text{ m}
\]

Therefore, \( \ell_1 - \ell_2 = 0.351 \text{ m} \). The wavelength of the sound is \( \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{1466 \text{ Hz}} = 0.234 \text{ m} \). Dividing the distance \( \ell_1 - \ell_2 \) by the wavelength \( \lambda \) gives the number of wavelengths in this distance:

\[
\text{Number of wavelengths} = \frac{\ell_1 - \ell_2}{\lambda} = \frac{0.351 \text{ m}}{0.233 \text{ m}} = 1.5
\]

Since the number of wavelengths is a half-integer number \( \left( 1\frac{1}{2} \right) \), destructive interference occurs at the listener.
b. The wavelength of the sound is now \( \lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{977 \text{ Hz}} = 0.351 \text{ m} \). Dividing the distance \( \ell_1 - \ell_2 \) by the wavelength \( \lambda \) gives the number of wavelengths in that distance:

\[
\text{Number of wavelengths} = \frac{\ell_1 - \ell_2}{\lambda} = \frac{0.351 \text{ m}}{0.351 \text{ m}} = 1
\]

Since the number of wavelengths is an integer number (1), [constructive interference] occurs at the listener.