CHAPTER 18 | *ELECTRIC FORCES AND ELECTRIC FIELDS*

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

---

1. 1.9×10^{13}

2. (b) Suppose that A is positive and B is negative. Since C and A also attract each other, C must be negative. Thus, B and C repel each other, because they have like charges (both negative). Suppose, however, that A is negative and B is positive. Since C and A also attract each other, C must be positive. Again we conclude that B and C repeal each other, because they have like charges (both positive).

3. (a) The ball is electrically neutral (net charge equals zero). However, it is made from a conducting material, so it contains electrons that are free to move. The rod attracts some of these (negative) electrons to the side of the ball nearest the rod, leaving the opposite side of the ball positively charged. Since the negative side of the ball is closer to the positive rod than the positive side, a net attractive force arises.

4. (d) The fact that the positive rod repels one object indicates that that object carries a net positive charge. The fact that the rod repels the other object indicates that that object carries a net negative charge. Since both objects are identical and made from conducting material, they share the combined net charges equally after they are touched together. Since the rod repels each object after they are touched, each object must then carry a net positive charge. But the net electric charge of any isolated system is conserved, so the total net charge initially must also have been positive. This means that the initial positive charge had the greater magnitude.

5. (c) This distribution is not possible because of the law of conservation of electric charge. The total charge on the three objects here is $\frac{9}{8}q$, whereas only q was present initially.

6. (c) This is an example of charging by induction. The negatively charged rod repels free electrons in the metal. These electrons move through the point of contact and into the sphere farthest away from the rod, giving it an induced charge of $-q$. The sphere nearest the rod acquires an induced charge of $+q$. As long as the rod is kept in place while the spheres are separated, these induced charges cannot recombine and remain on the spheres.

7. (b) Coulomb's law states that the magnitude of the force is given by $F = k\frac{|q_1||q_2|}{r^2}$. Doubling the magnitude of each charge as in A would increase the numerator by a factor of four, but this is offset by the change in separation, which increases the denominator by a factor of $2^2 = 4$. Doubling the magnitude of only one charge as in D would increase the numerator by

a factor of two, but this is offset by the change in separation, which increases the denominator by a factor of $\left(\sqrt{2}\right)^2 = 2$.

8. (e) Coulomb's law states that the magnitude of the force is given by $F = k\frac{|q_1||q_2|}{r^2}$. The force is directed along the line between the charges and is an attraction for unlike charges and a repulsion for like charges. Charge B is attracted by charge A with a force of magnitude $k\frac{|q||q|}{d^2}$ and repelled by charge C with a force of the same magnitude. Since both forces point to the left, the net force acting on B has a magnitude of $2k\frac{|q||q|}{d^2}$. Charge A is attracted by charge B with a force of $k\frac{|q||q|}{d^2}$ and also by charge C with a force of $k\frac{|q||q|}{(2d)^2}$. Since both forces point to the right, the net force acting on A has a magnitude of $(1.25)k\frac{|q||q|}{d^2}$. Charge C is pushed to the right by B with a force of $k\frac{|q||q|}{d^2}$ and pulled to the left by A with a force of $k\frac{|q||q|}{(2d)^2}$. Since these two forces have different directions, the net force acting on C has a magnitude of $(0.75)k\frac{|q||q|}{d^2}$.

9. (b) According to Coulomb's law, the magnitude of the force that any one of the point charges exerts on another point charge is given by $F = k\frac{|q||q|}{d^2}$, where d is the length of each side of the triangle. The charge at B experiences a repulsive force from the charge at A and an attractive force from the charge at C. Both forces have vertical components, but one points in the $+y$ direction and the other in the $-y$ direction. These vertical components have equal magnitudes and cancel, leaving a resultant that is parallel to the x axis.

10. 8.5 μC

11. (e) According to Equation 18.2, the force exerted on a charge by an electric field is proportional to the magnitude of the charge. Since the negative charge has twice the magnitude of the positive charge, the negative charge experiences twice the force. Furthermore, the direction of the force on the positive charge is in the same direction as the field, so that we can conclude that the field points due west. The force on the negative charge points opposite to the field and, therefore, points due east.

12. (c) The electric field created by a point charge has a magnitude $E = \frac{k|q|}{r^2}$ and is inversely proportional to the square of the distance r. If r doubles, the charge magnitude must increase by a factor of $2^2 = 4$ to keep the field the same.

13. (b) To the left of the positive charge the two contributions to the total field have opposite directions. There is a spot in this region at which the field from the smaller, but closer, positive charge exactly offsets the field from the greater, but more distant, negative charge.

14. (e) Consider the charges on opposite corners. In all of the arrangements these are like charges. This means that the two field contributions created at the center of the square point in opposite directions and, therefore, cancel. Thus, only the charge opposite the empty corner determines the magnitude of the net field at the center of the square. Since the point charges all have the same magnitude, the net field there has the same magnitude in each arrangement.

15. 1.8×10^{-6} C/m^2

16. (c) The tangent to the field line gives the direction of the electric field at a point. At A the tangent points due south, at B southeast, and at C due east.

17. (a) The electric field has a greater magnitude where the field lines are closer together. They are closest together at B and farthest apart at A. Therefore, the field has the greatest magnitude at B and the smallest magnitude at A.

18. (d) In a conductor electric charges can readily move in response to an electric field. In A, B, and C the electric charges experience an electric field and, hence, a force from neighboring charges and will move outward, away from each other. They will rearrange themselves so that the electric field within the metal is zero at equilibrium. This means that they will reside on the outermost surface. Thus, only D could represent charges in equilibrium.

19. 1.3 N·m^2/C

20. 0.45 N·m^2/C

CHAPTER 18 *ELECTRIC FORCES AND ELECTRIC FIELDS*

PROBLEMS

---

10. ***REASONING***

a. The magnitude of the electrostatic force that acts on each sphere is given by Coulomb's law as $F = k|q_1||q_2|/r^2$, where $|q_1|$ and $|q_2|$ are the magnitudes of the charges, and r is the distance between the centers of the spheres.

b. When the spheres are brought into contact, the net charge after contact and separation must be equal to the net charge before contact. Since the spheres are identical, the charge on each after being separated is one-half the net charge. Coulomb's law can be applied again to determine the magnitude of the electrostatic force that each sphere experiences.

SOLUTION

a. The magnitude of the force that each sphere experiences is given by Coulomb's law as:

$$F = \frac{k|q_1||q_2|}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(20.0\times10^{-6}\ \text{C}\right)\left(50.0\times10^{-6}\ \text{C}\right)}{\left(2.50\times10^{-2}\ \text{m}\right)^2} = \boxed{1.44\times10^4\ \text{N}}$$

Because the charges have opposite signs, the force is $\boxed{\text{attractive}}$.

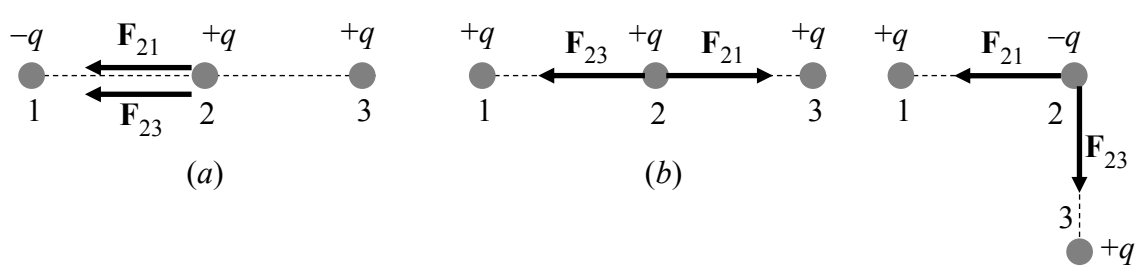
b. The net charge on the spheres is $-20.0\ \mu\text{C} + 50.0\ \mu\text{C} = +30.0\ \mu\text{C}$. When the spheres are brought into contact, the net charge after contact and separation must be equal to the net charge before contact, or $+30.0\ \mu\text{C}$. Since the spheres are identical, the charge on each after being separated is one-half the net charge, so $q_1 = q_2 = +15.0\ \mu\text{C}$. The electrostatic force that acts on each sphere is now

$$F = \frac{k|q_1||q_2|}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(15.0\times10^{-6}\ \text{C}\right)\left(15.0\times10^{-6}\ \text{C}\right)}{\left(2.50\times10^{-2}\ \text{m}\right)^2} = \boxed{3.24\times10^3\ \text{N}}$$

Since the charges now have the same signs, the force is $\boxed{\text{repulsive}}$.

---

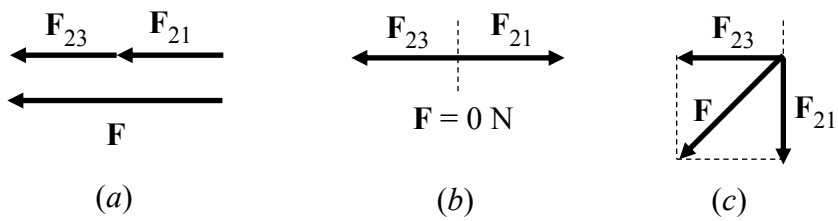
14. ***REASONING*** The electrical force that each charge exerts on charge 2 is shown in the following drawings. $\mathbf{F}_{21}$ is the force exerted on 2 by 1, and $\mathbf{F}_{23}$ is the force exerted on 2



(c)

by 3. Each force has the same magnitude, because the charges have the same magnitude and the distances are equal.

The net electric force **F** that acts on charge 2 is shown in the following diagrams.



(*a*) (*b*) (*c*)

It can be seen from the diagrams that the largest electric force occurs in (a), followed by (c), and then by (b).

SOLUTION The magnitude F_{21} of the force exerted on 2 by 1 is the same as the magnitude F_{23} of the force exerted on 2 by 3, since the magnitudes of the charges are the same and the distances are the same. Coulomb's law gives the magnitudes as

$$F_{21} = F_{23} = \frac{k|q||q|}{r^2}$$

$$= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.6\times10^{-6}\ \text{C}\right)\left(8.6\times10^{-6}\ \text{C}\right)}{\left(3.8\times10^{-3}\ \text{m}\right)^2} = 4.6\times10^4\ \text{N}$$

In part (a) of the drawing showing the net electric force acting on charge 2, both $\mathbf{F}_{21}$ and $\mathbf{F}_{23}$ point to the left, so the net force has a magnitude of

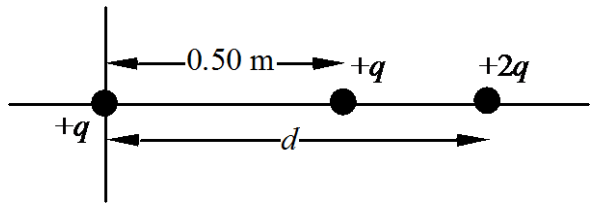
$$F = 2F_{12} = 2\left(4.6\times10^4\ \text{N}\right) = \boxed{9.2\times10^4\ \text{N}}$$

In part (b) of the drawing showing the net electric force acting on charge 2, $\mathbf{F}_{21}$ and $\mathbf{F}_{23}$ point in opposite directions, so the net force has a magnitude of $\boxed{0\ \text{N}}$.

In part (c) showing the net electric force acting on charge 2, the magnitude of the net force can be obtained from the Pythagorean theorem:

$$F = \sqrt{F_{21}^2 + F_{23}^2} = \sqrt{\left(4.6\times10^4\ \text{N}\right)^2 + \left(4.6\times10^4\ \text{N}\right)^2} = \boxed{6.5\times10^4\ \text{N}}$$

16. ***REASONING*** The drawing at the right shows the set-up. The force on the $+q$ charge at the origin due to the other $+q$ charge is given by Coulomb's law (Equation 18.1), as is the force due to the $+2q$ charge. These two forces point to the left, since each is repulsive. The sum of the two is twice the force on the $+q$ charge at the origin due to the other $+q$ charge alone.



SOLUTION Applying Coulomb's law, we have

$$\underbrace{\frac{k|q||q|}{(0.50\text{ m})^2}}_{\substack{\text{Force due to } +q \\ \text{charge at } x=+0.50\text{ m}}} + \underbrace{\frac{k|2q||q|}{(d)^2}}_{\substack{\text{Force due to } +2q \\ \text{charge at } x=+d}} = \underbrace{2\frac{k|q||q|}{(0.50\text{ m})^2}}_{\substack{\text{Twice the force due to} \\ +q \text{ charge at } x=+0.50\text{ m}}}$$

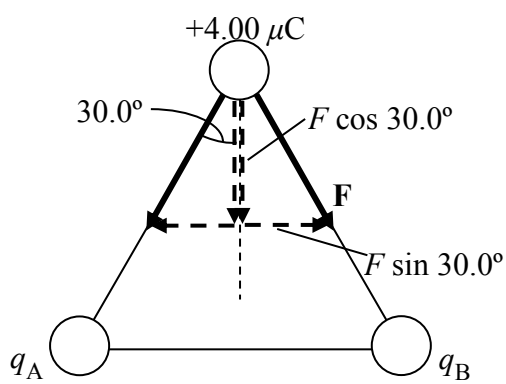
Rearranging this result and solving for d give

$$\frac{k|2q||q|}{(d)^2} = \frac{k|q||q|}{(0.50\text{ m})^2} \qquad \text{or} \qquad d^2 = 2(0.50\text{ m})^2 \qquad \text{or} \qquad d = \pm 0.71\text{ m}$$

We reject the negative root, because a negative value for d would locate the $+2q$ charge to the left of the origin. Then, the two forces acting on the charge at the origin would have different directions, contrary to the statement of the problem. Therefore, the $+2q$ charge is located at a position of $\boxed{x = +0.71\text{ m}}$.

---

20. ***REASONING*** The unknown charges can be determined using Coulomb's law to express the electrostatic force that each unknown charge exerts on the 4.00 μC charge. In applying this law, we will use the fact that the net force points downward in the drawing. This tells us that the unknown charges are both negative and have the same magnitude, as can be understood with the help of the free-body diagram for the 4.00 μC charge that is shown at the right. The diagram shows the attractive force **F** from each negative charge directed along the lines between the charges. Only when each force has the same magnitude (which is the case when both unknown charges have the same magnitude) will the resultant force point vertically downward. This occurs because the horizontal components of the forces cancel, one pointing to the right and the other to the left (see the diagram). The vertical components reinforce to give the observed downward net force.



SOLUTION Since we know from the ***REASONING*** that the unknown charges have the same magnitude, we can write Coulomb's law as follows:

$$F = k\frac{\left(4.00\times10^{-6}\ \text{C}\right)\left|q_{\text{A}}\right|}{r^2} = k\frac{\left(4.00\times10^{-6}\ \text{C}\right)\left|q_{\text{B}}\right|}{r^2}$$

The magnitude of the net force acting on the 4.00 μC charge, then, is the sum of the magnitudes of the two vertical components F cos 30.0º shown in the free-body diagram:

$$\Sigma F = k\frac{\left(4.00\times10^{-6}\ \text{C}\right)\left|q_{\text{A}}\right|}{r^2}\cos 30.0° + k\frac{\left(4.00\times10^{-6}\ \text{C}\right)\left|q_{\text{B}}\right|}{r^2}\cos 30.0°$$

$$= 2k\frac{\left(4.00\times10^{-6}\ \text{C}\right)\left|q_{\text{A}}\right|}{r^2}\cos 30.0°$$

Solving for the magnitude of the charge gives

$$\left|q_{\text{A}}\right| = \frac{\left(\Sigma F\right)r^2}{2k\left(4.00\times10^{-6}\ \text{C}\right)\cos 30.0°}$$

$$= \frac{\left(405\ \text{N}\right)\left(0.0200\ \text{m}\right)^2}{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00\times10^{-6}\ \text{C}\right)\cos 30.0°} = 2.60\times10^{-6}\ \text{C}$$

Thus, we have $q_{\text{A}} = q_{\text{B}} = \boxed{-2.60\times10^{-6}\ \text{C}}$.

---

27. **SSM** ***REASONING*** The charged insulator experiences an electric force due to the presence of the charged sphere shown in the drawing in the text. The forces acting on the insulator are the downward force of gravity (i.e., its weight, $W = mg$), the electrostatic force $F = k\left|q_1\right|\left|q_2\right|/r^2$ (see Coulomb's law, Equation 18.1) pulling to the right, and the tension T in the thread pulling up and to the left at an angle θ with respect to the vertical, as shown in the drawing in the problem statement. We can analyze the forces to determine the desired quantities θ and T.

SOLUTION.

a. We can see from the diagram given with the problem statement that

$$T_x = F \qquad \text{which gives} \qquad T\sin\theta = k\left|q_1\right|\left|q_2\right|/r^2$$

and

$$T_y = W \qquad \text{which gives} \qquad T\cos\theta = mg$$

Dividing the first equation by the second yields

$$\frac{T\sin\theta}{T\cos\theta} = \tan\theta = \frac{k|q_1||q_2|/r^2}{mg}$$

Solving for θ, we find that

$$\theta = \tan^{-1}\left(\frac{k|q_1||q_2|}{mgr^2}\right)$$

$$= \tan^{-1}\left[\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.600\times10^{-6}\ \text{C})(0.900\times10^{-6}\ \text{C})}{(8.00\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.150\ \text{m})^2}\right] = \boxed{15.4°}$$

b. Since $T\cos\theta = mg$, the tension can be obtained as follows:

$$T = \frac{mg}{\cos\theta} = \frac{(8.00\times10^{-2}\ \text{kg})\ (9.80\ \text{m/s}^2)}{\cos 15.4°} = \boxed{0.813\ \text{N}}$$

---

31. ***SOLUTION*** Knowing the electric field at a spot allows us to calculate the force that acts on a charge placed at that spot, without knowing the nature of the object producing the field. This is possible because the electric field is defined as $\mathbf{E} = \mathbf{F}/q_0$, according to Equation 18.2. This equation can be solved directly for the force $\mathbf{F}$, if the field $\mathbf{E}$ and charge q_0 are known.

SOLUTION Using Equation 18.2, we find that the force has a magnitude of

$$F = E|q_0| = (260\,000\ \text{N/C})\left(7.0\times10^{-6}\ \text{C}\right) = 1.8\ \text{N}$$

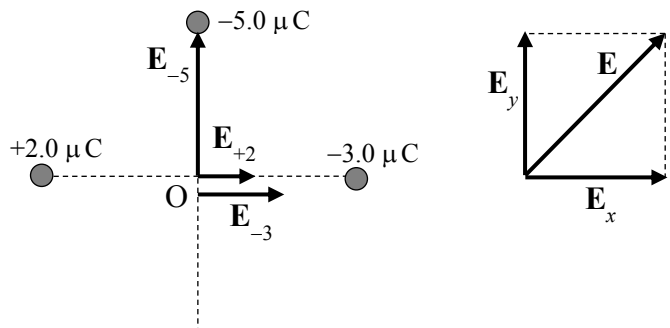
If the charge were positive, the direction of the force would be due west, the same as the direction of the field. But the charge is negative, so the force points in the opposite direction or due east. Thus, the force on the charge is $\boxed{1.8\ \text{N due east}}$.

---

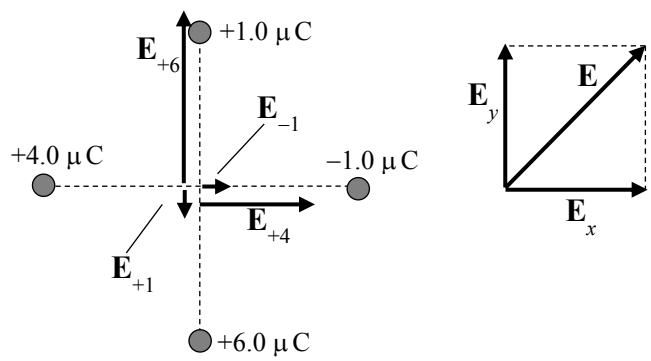
34. ***REASONING***

Part (*a*) of the drawing given in the text. The electric field produced by a charge points away from a positive charge and toward a negative charge. Therefore, the electric field $\mathbf{E}_{+2}$ produced by the +2.0 μC charge points away from it, and the electric fields $\mathbf{E}_{-3}$ and $\mathbf{E}_{-5}$ produced by the –3.0 μC and –5.0 μC charges point toward them (see the left-hand side of the following drawing). The magnitude of the electric field produced by a point charge is given by Equation 18.3 as $E = k|q|/r^2$. Since the distance from each charge to the origin is the same, the magnitude of the electric field is proportional only to the magnitude $|q|$ of the charge. Thus, the x component $\mathbf{E}_x$ of the net electric field is proportional to

5.0 μC (2.0 μC + 3.0 μC). Since only one of the charges produces an electric field in the y direction, the y component $\mathbf{E}_y$ of the net electric field is proportional to the magnitude of this charge, or 5.0 μC. Thus, the x and y components are equal, as indicated at the right-hand side of the following drawing, where the net electric field **E** is also shown.



Part (*b*) of the drawing given in the text. Using the same arguments as earlier, we find that the electric fields produced by the four charges are shown at the left-hand side of the following drawing. These fields also produce the same net electric field **E** as before, as indicated at the right-hand side of the following drawing.



SOLUTION

Part (*a*) of the drawing given in the text. The net electric field in the x direction is

$$E_x = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(2.0\times10^{-6}\ \mathrm{C}\right)}{\left(0.061\,\mathrm{m}\right)^2} + \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(3.0\times10^{-6}\ \mathrm{C}\right)}{\left(0.061\,\mathrm{m}\right)^2}$$

$$= 1.2\times10^7\ \mathrm{N/C}$$

The net electric field in the y direction is

$$E_y = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(5.0\times10^{-6}\ \mathrm{C}\right)}{\left(0.061\,\mathrm{m}\right)^2} = 1.2\times10^7\ \mathrm{N/C}$$

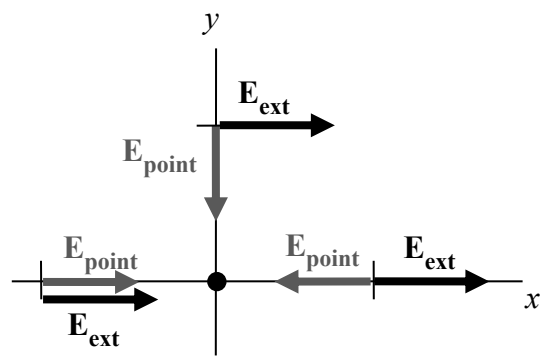
The magnitude of the net electric field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(1.2\times10^7\ \text{N/C}\right)^2 + \left(1.2\times10^7\ \text{N/C}\right)^2} = \boxed{1.7\times10^7\ \text{N/C}}$$

Part (*b*) of the drawing given in the text. The magnitude of the net electric field is the same as determined for part (a); $E = \boxed{1.7\times10^7\ \text{N/C}}$.

---

43. ***REASONING*** The electric field is given by Equation 18.2 as the force **F** that acts on a test charge q_0, divided by q_0. Although the force is not known, the acceleration and mass of the charged object are given. Therefore, we can use Newton's second law to determine the force as the mass times the acceleration and then determine the magnitude of the field directly from Equation 18.2. The force has the same direction as the acceleration. The direction of the field, however, is in the direction opposite to that of the acceleration and force. This is because the object carries a negative charge, while the field has the same direction as the force acting on a positive test charge.



SOLUTION According to Equation 18.2, the magnitude of the electric field is

$$E = \frac{F}{|q_0|}$$

According to Newton's second law, the net force acting on an object of mass m and acceleration a is $\Sigma F = ma$. Here, the net force is the electrostatic force F, since that force alone acts on the object. Thus, the magnitude of the electric field is

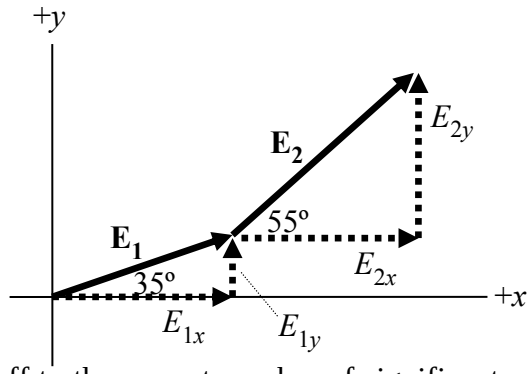
$$E = \frac{F}{|q_0|} = \frac{ma}{|q_0|} = \frac{\left(3.0\times10^{-3}\ \text{kg}\right)\left(2.5\times10^3\ \text{m/s}^2\right)}{34\times10^{-6}\ \text{C}} = \boxed{2.2\times10^5\ \text{N/C}}$$

The direction of this field is opposite to the direction of the acceleration. Thus, the field points $\boxed{\text{along the } -x \text{ axis}}$.

---

46. ***REASONING*** The electric field is a vector. Therefore, the total field **E** is the vector sum of its two parts, or $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$. We will carry out this vector addition by using the method of components (see Section 1.8).

SOLUTION The drawing at the right shows the two vectors $\mathbf{E_1}$ and $\mathbf{E_2}$, together with their x and y components. In the following table, we calculate the components of each vector. We also show the x component E_x of the total field as the sum of the individual x components of $\mathbf{E_1}$ and $\mathbf{E_2}$ and the y component E_y of the total field as the sum of the individual y components of $\mathbf{E_1}$ and $\mathbf{E_2}$. Note that the calculations in the table carry additional significant figures. Rounding off to the correct number of significant figures will be done when we calculate the final answers.



Vector	x component	y component
$\mathbf{E}_1$	$E_{1x} = E_1 \cos\theta_1 = (1200\text{ N/C})\cos 35°$ $= 983\text{ N/C}$	$E_{1y} = E_1 \sin\theta_1 = (1200\text{ N/C})\sin 35°$ $= 688\text{ N/C}$
$\mathbf{E}_2$	$E_{2x} = E_2 \cos\theta_2 = (1700\text{ N/C})\cos 55°$ $= 975\text{ N/C}$	$E_{2y} = E_2 \sin\theta_2 = (1700\text{ N/C})\sin 55°$ $= 1393\text{ N/C}$
$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$	$E_x = E_{1x} + E_{2x} = 983\text{ N/C} + 975\text{ N/C}$ $= 1958\text{ N/C}$	$E_y = E_{1y} + E_{2y} = 688\text{ N/C} + 1393\text{ N/C}$ $= 2081\text{ N/C}$

Since the components E_x and E_y of the total field are perpendicular, we can use the Pythagorean theorem to calculate the magnitude E of the total field and trigonometry to calculate the directional angle θ:

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(1958\text{ N/C})^2 + (2081\text{ N/C})^2} = \boxed{2900\text{ N/C}}$$

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{2081\text{ N/C}}{1958\text{ N/C}}\right) = \boxed{47°}$$

---

53. ***REASONING AND SOLUTION*** Since the thread makes an angle of 30.0° with the vertical, it can be seen that the electric force on the ball, F_e, and the gravitational force, mg, are related by

$$F_e = mg \tan 30.0°$$

The force F_e is due to the charged ball being in the electric field of the parallel plate capacitor. That is,

$$F_e = E\left|q_{\text{ball}}\right| \qquad (1)$$

where $|q_{\text{ball}}|$ is the magnitude of the ball's charge and E is the magnitude of the field due to the plates. According to Equation 18.4 E is

$$E = \frac{q}{\varepsilon_0 A} \tag{18.4}$$

where q is the magnitude of the charge on each plate and A is the area of each plate. Substituting Equation 18.4 into Equation (1) gives

$$F_{\text{e}} = mg \tan 30.0° = \frac{q|q_{\text{ball}}|}{\varepsilon_0 A}$$

Solving for q yields

$$q = \frac{\varepsilon_0 Amg \tan 30.0°}{|q_{\text{ball}}|}$$

$$= \frac{\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right]\left(0.0150\ \text{m}^2\right)\left(6.50\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\tan 30.0°}{0.150\times10^{-6}\ \text{C}}$$

$$= \boxed{3.25\times10^{-8}\ \text{C}}$$

---

54. ***REASONING AND SOLUTION*** Gauss' Law is given by text Equation 18.7: $\Phi_{\text{E}} = \dfrac{Q}{\varepsilon_0}$, where Q is the *net charge* enclosed by the Gaussian surface.

a. $\Phi_{\text{E}} = \dfrac{3.5\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = \boxed{4.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$

b. $\Phi_{\text{E}} = \dfrac{-2.3\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = \boxed{-2.6\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$

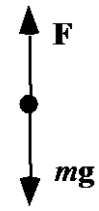
c. $\Phi_{\text{E}} = \dfrac{(3.5\times10^{-6}\ \text{C})+(-2.3\times10^{-6}\ \text{C})}{8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = \boxed{1.4\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$

---

66. ***REASONING*** Since the charged droplet (charge $= q$) is suspended motionless in the electric field **E**, the net force on the droplet must be zero. There are two forces that act on the droplet, the force of gravity $\mathbf{W} = m\mathbf{g}$, and the electric force $\mathbf{F} = q\mathbf{E}$ due to the electric field. Since the net force on the droplet is zero, we conclude that $mg = |q|E$. We can use this reasoning to determine the sign and the magnitude of the charge on the droplet.

SOLUTION

a. Since the net force on the droplet is zero, and the weight of magnitude W points downward, the electric force of magnitude $F = |q|E$ must point upward. Since the electric field points upward, the excess charge on the droplet must be $\boxed{\text{positive}}$ in order for the force **F** to point upward.



b. Using the expression $mg = |q|E$, we find that the magnitude of the excess charge on the droplet is

$$|q| = \frac{mg}{E} = \frac{(3.50\times10^{-9}\ \text{kg})(9.80\ \text{m/s}^2)}{8480\ \text{N/C}} = 4.04\times10^{-12}\ \text{C}$$

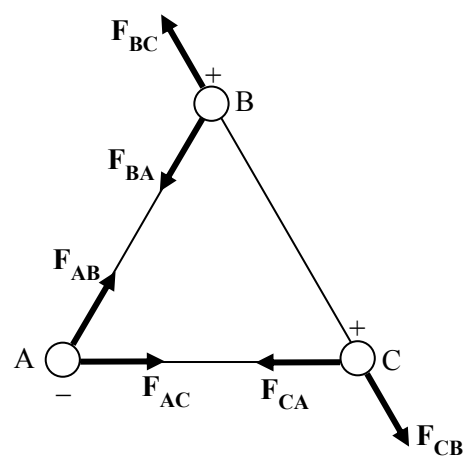
The charge on a proton is 1.60×10^{-19} C, so the excess number of protons is

$$\left(4.04\times10^{-12}\ \text{C}\right)\left(\frac{1\ \text{proton}}{1.60\times10^{-19}\ \text{C}}\right) = \boxed{2.53\times10^{7}\ \text{protons}}$$

---

72. ***REASONING***

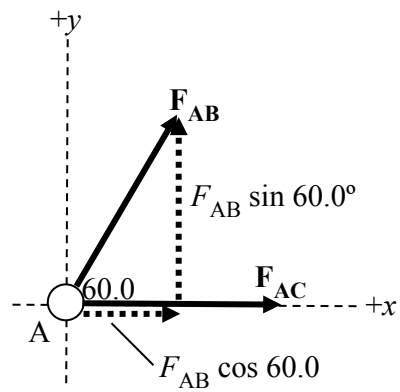
The drawing at the right shows the forces that act on the charges at each corner. For example, $\mathbf{F}_{AB}$ is the force exerted on the charge at corner A by the charge at corner B. The directions of the forces are consistent with the fact that like charges repel and unlike charges attract. Coulomb's law indicates that all of the forces shown have the same magnitude, namely, $F = k|q|^2/L^2$, where $|q|$ is the magnitude of each of the charges and L is the length of each side of the equilateral triangle. The magnitude is the same for each force, because $|q|$ and L are the same for each pair of charges.



The net force acting at each corner is the sum of the two force vectors shown in the drawing, and the net force is greatest at corner A. This is because the angle between the two vectors at A is 60º. With the angle less than 90º, the two vectors partially reinforce one another. In comparison, the angles between the vectors at corners B and C are both 120º, which means that the vectors at those corners partially offset one another.

The net forces acting at corners B and C have the same magnitude, since the magnitudes of the individual vectors are the same and the angles between the vectors at both B and C are the same (120º). Thus, vector addition by either the tail-to-head method (see Section 1.6) or the component method (see Section 1.8) will give resultant vectors that have different directions but the same magnitude. The magnitude of the net force is the smallest at these two corners.

SOLUTION REASONING, As pointed out in the the magnitude of any individual force vector is $F = k|q|^2 / L^2$. With this in mind, we apply the component method for vector addition to the forces at corner A, which are shown in the drawing at the right, together with the appropriate components. The x component ΣF_x and the y component ΣF_y of the net force are



$$\left(\Sigma F_x\right)_{\mathrm{A}} = F_{\mathrm{AB}} \cos 60.0° + F_{\mathrm{AC}} = F\left(\cos 60.0° + 1\right)$$

$$\left(\Sigma F_y\right)_{\mathrm{A}} = F_{\mathrm{AB}} \sin 60.0° = F \sin 60.0°$$

where we have used the fact that $F_{\mathrm{AB}} = F_{\mathrm{AC}} = F$. The Pythagorean theorem indicates that the magnitude of the net force at corner A is

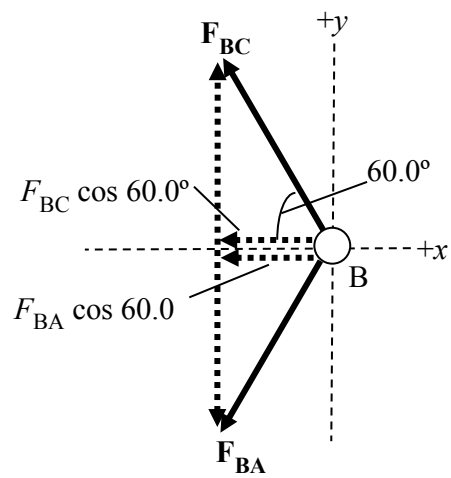
$$\left(\Sigma F\right)_{\mathrm{A}} = \sqrt{\left(\Sigma F_x\right)_{\mathrm{A}}^2 + \left(\Sigma F_y\right)_{\mathrm{A}}^2} = \sqrt{F^2\left(\cos 60.0° + 1\right)^2 + \left(F \sin 60.0°\right)^2}$$

$$= F\sqrt{\left(\cos 60.0° + 1\right)^2 + \left(\sin 60.0°\right)^2} = k\frac{|q|^2}{L^2}\sqrt{\left(\cos 60.0° + 1\right)^2 + \left(\sin 60.0°\right)^2}$$

$$= \left(8.99 \times 10^9\ \mathrm{N \cdot m^2 / C^2}\right)\frac{\left(5.0 \times 10^{-6}\ \mathrm{C}\right)^2}{\left(0.030\ \mathrm{m}\right)^2}\sqrt{\left(\cos 60.0° + 1\right)^2 + \left(\sin 60.0°\right)^2}$$

$$= \boxed{430\ \mathrm{N}}$$

We now apply the component method for vector addition to the forces at corner B. These forces, together with the appropriate components are shown in the drawing at the right. We note immediately that the two vertical components cancel, since they have opposite directions. The two horizontal components, in contrast, reinforce since they have the same direction. Thus, we have the following components for the net force at corner B:



$$\left(\Sigma F_x\right)_{\mathrm{B}} = -F_{\mathrm{BC}}\cos 60.0° - F_{\mathrm{BA}}\cos 60.0° = -2F\cos 60.0°$$

$$\left(\Sigma F_y\right)_{\mathrm{B}} = 0$$

where we have used the fact that $F_{\mathrm{BC}} = F_{\mathrm{BA}} = F$. The Pythagorean theorem indicates that the magnitude of the net force at corner B is

$$\left(\Sigma F\right)_{\mathrm{B}} = \sqrt{\left(\Sigma F_x\right)_{\mathrm{B}}^2 + \left(\Sigma F_y\right)_{\mathrm{B}}^2} = \sqrt{\left(-2F\cos 60.0°\right)^2 + \left(0\right)^2} = 2F\cos 60.0°$$

$$= 2k\frac{|q|^2}{L^2}\cos 60.0° = 2\left(8.99\times 10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(5.0\times 10^{-6}\ \mathrm{C}\right)^2}{\left(0.030\ \mathrm{m}\right)^2}\cos 60.0°$$

$$= \boxed{250\ \mathrm{N}}$$

As discussed in the ***REASONING***, the magnitude of the net force acting on the charge at corner C is the same as that acting on the charge at corner B, so $\left(\Sigma F\right)_{\mathrm{C}} = \boxed{250\ \mathrm{N}}$.